Combining reconstruction and edge detection in CT

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Problem and Goal

Edge detection in computed tomography (CT) scans is a challenging task as the underlying image reconstruction problem is ill-posed. Besides noise amplification, classical reconstruction algorithms may generate additional subsampling artifacts, which leads to unreliable edge maps.

We present two methods that allow for a stable reconstruction of edges directly from CT data. In particular, we show that iterative sparse regularization is able to reconstruct a gradient image from the data reliably, which enables the subsequent application of the Canny algorithm.

Mathematical model

The mathematical model of CT is given by the *Radon transform* of a function $f\colon \mathbb{R}^2 \to \mathbb{R}$, which is defined as

$$\mathcal{R}f(\varphi, s) := \int_{\mathbb{R}} f(s\theta(\varphi) + t\theta^{\perp}(\varphi))\,\mathrm{d}t,$$

where $\theta \in \mathbb{S}^1$ is a unit vector depending on an angle $\varphi \in [0, \pi)$, and $s \in \mathbb{R}$ is the singed distance from the origin. The value $\mathcal{R}f(\varphi, s)$ represents one x-ray measurement along the line $L(\varphi, s) = \{s\theta(\varphi)) + t\theta^{\perp}(\varphi) : t \in \mathbb{R}\}$.

The reconstruction problem is given by

$$\mathcal{R}f_{\varepsilon} = g,$$

with $f_{\varepsilon} := f * g_{\varepsilon}$, where

$$g_{\varepsilon}(x) := \frac{1}{2\pi\varepsilon^2} \exp\left(-\frac{\|x\|^2}{2\varepsilon^2}\right), \quad \varepsilon > 0.$$

Subsequently, we aim to reconstruct the gradient images

$$\frac{\partial f_{\varepsilon}}{\partial x_j} = \frac{\partial}{\partial x_j}(f * g_{\varepsilon}) = f * \frac{\partial g_{\varepsilon}}{\partial x_j}. \tag{1}$$

where $j \in \{1, 2\}$ denotes the gradient in x_1 and x_2 direction, respectively.

Methods

We propose two methods as reconstruction strategies:

1. FBP-type approach

Let $W_{\varepsilon,j}$ be the function of two variables (φ, s) which is defined in the Fourier domain by

$$\widehat{W}_{\varepsilon,j}(\varphi, \omega) := \frac{1}{4\pi} \cdot \theta_j(\varphi) \cdot i \cdot \omega \cdot |\omega| \cdot \exp\left(\frac{-\varepsilon^2\omega^2}{2}\right),$$

where the Fourier transform is applied with respect to the s-variable. Then (1) is given by

$$\frac{\partial f_{\varepsilon}}{\partial x_j} = \mathcal{B}(\mathcal{R}f *_s W_{\varepsilon,j}), \tag{2}$$

where $\mathcal{B}$ is the backprojection operator for the Radon transform.

2. Sparse Regularization

To account for the negative effects of undersampling of CT data, we propose to use an iterative reconstruction method. We calculate the derivatives (1) approximately via:

$$\mathbf{f}^{(j)}_{\lambda,\varepsilon} = \arg\min_{\mathbf{f}} \|\mathbf{Rf} - \mathbf{y} * G_{\varepsilon,j}\|_2^2 + \lambda \|\mathbf{f}\|_1, \tag{3}$$

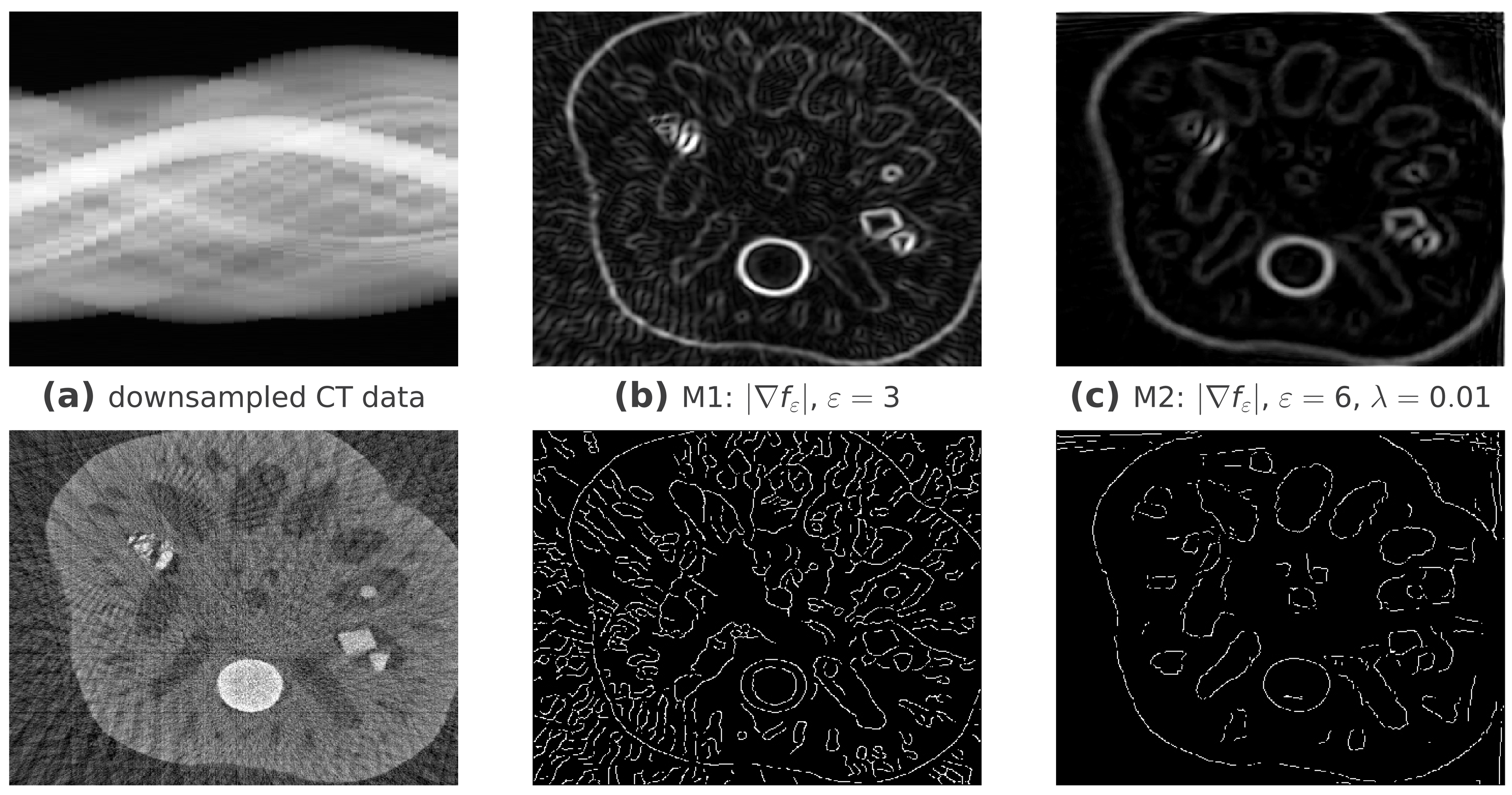
The bold face symbols denote the discretized versions of the corresponding continuous objects. To solve this minimization problem, we used the well-known ISTA algorithm (cf. [2]).

Results

For simulation, we used real CT data of a lotus root, cf. [1]. To simulate angular undersampling we downsampled this data to 738 equispaced samples in the s-variable and 36 evenly distributed angles in $[0, \pi)$. During experiments, parameters where chosen based on visual inspection of edge detection results.

Figure 1d clearly shows FBP reconstruction errors due to under sampled data (Figure 1a). Directly applying gradient methods for edge detection, would lead to an uninterpretable image.

The middle column shows reconstruction using (2). Clearly method 1 cannot compensate for the undersampling artifacts. On the right, ℓ^1-regularization successfully reduces the number of artifacts and detects the edges more reliably.



(a) downsampled CT data **(b)** M1: $|\nabla f_{\varepsilon}|$, $\varepsilon = 3$ **(c)** M2: $|\nabla f_{\varepsilon}|$, $\varepsilon = 6$, $\lambda = 0.01$

(d) FBP reconstruction **(e)** M1: edge map (Canny) **(f)** M2 2: edge map (Canny)

Figure 1: Rebinned CT data of a lotus root (cf. [1]) from 36 evenly distributed angles (a) and FBP reconstruction (d). The images of the gradient magnitude $|\nabla f_{\varepsilon}|$ are shown in (b) and (c), and the corresponding edge detection results using the Canny algorithm in (e) and (f).

In other experiments we observed that method 2 outperforms the method 1 whenever the CT data was not sampled properly. For dense angular sampling, we found that both methods produce similar edge detection results.

Summary and Outlook

The proposed methods produce promising results. As expected, method 1 performed very well for fully sampled data, but struggled in the incomplete data case. For heavily undersampled Radon data, method 2 still was able to produce a gradient image reliably.

Further research may be conducted in terms of optimal parameter selection as well as edge detection methods involving the Laplacian-of-Gaussian-type algorithms.

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